

Write your homework *neatly, in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. When appropriate, *write in sentences*.

The main theory of section 4.3 is summarized below.

Definition 1. Let f be a function defined on an interval I .

We say that f is *increasing on I* if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$.

We say that f is *decreasing on I* if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$.

The following is another corollary to the Mean Value Theorem.

Corollary 3. Suppose that f is continuous on $[a, b]$ and differentiable on (a, b) .

If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on $[a, b]$.

If $f'(x) < 0$ for all $x \in (a, b)$, then f is decreasing on $[a, b]$.

Theorem 1. (First Derivative Test)

Suppose that f is differentiable in on interval containing c , and that $f'(c) = 0$.

- If f' changes sign from positive to negative at c , then f has a local maximum at c .
- If f' changes sign from negative to positive at c , then f has a local minimum at c .
- If f' does not change sign at c , then f does not have a local extremum at c .

Problem 1 (Thomas §4.3 # 1). Let

$$f'(x) = x(x - 1).$$

What are the critical points of f ? On what intervals is f increasing or decreasing? At points, if any, does f attain local maximum or minimum values?

Problem 2 (Thomas §4.3 # 4). Let

$$f'(x) = (x - 1)^2(x + 2)^2.$$

What are the critical points of f ? On what intervals is f increasing or decreasing? At points, if any, does f attain local maximum or minimum values?

Problem 3 (Thomas §4.3 # 12). Let

$$h(x) = 2x^3 - 18x.$$

Find the intervals on which the function is increasing or decreasing. Identify the points in the domain where h has a local extremum, and find the local extreme value. Which, if any, of the extreme values are absolute? Sketch the graph of h .

Problem 4 (Thomas §4.3 # 21). Let

$$g(x) = x\sqrt{8 - x^2}.$$

Find the intervals on which the function is increasing or decreasing. Identify the points in the domain where g has a local extremum, and find the local extreme value. Which, if any, of the extreme values are absolute?

Problem 5 (Thomas §4.3 # 48). Find the intervals on which $f(x) = ax^2 + bx + c$ ($a \neq 0$) is increasing and decreasing. Describe the reasoning behind your answer.

Here is another definition we need.

Definition 2. Let f be continuous on $[a, b]$. The *average rate of change* of f on $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a}.$$

This is the slope of the line from $(a, f(a))$ to $(b, f(b))$.

Problem 6. Let

$$f(x) = \frac{x^2 + 1}{x}.$$

- (a) Let $g(b)$ denote the average rate of change of f on $[1, b]$. Write $g(b)$ as a function of b . Simplify.
- (b) Find $\lim_{b \rightarrow \infty} g(b)$.

Problem 7. Find

$$\int x \tan^2(x^2) dx.$$

Problem 8. Find

$$\int x \cos(x) \sqrt{\sin(x^2)} dx.$$

Problem 9. Let

$$f(x) = x^3 - 6x^2 + 9x.$$

- (a) Find the zeros of f .
- (b) Find the zeros of f' .
- (c) Find the points on the graph of f where f has local extreme values.
- (d) Use this information to sketch the graph of f .

Problem 10. The Three Great VT's are the Intermediate Value Theorem (IVT), the Extreme Value Theorem (EVT), and the Mean Value Theorem (MVT).

- (a) Precisely write the hypothesis and conclusion of each of these theorems.
- (b) Match the following physical situation with the given theorem.
 - A motorist passed a policeman at 10:15 AM and passed his partner 10 miles down the road at 10:25. He was arrested for speeding at 60 miles per hour.
 - The chicken had to cross the road to get to the other side.
 - A man was born weighing 7 pounds, became a rich glutton, but died of starvation; yet, at some point in his life, he attained a maximum weight.